CS 383
HW 8

Due in class Wednesday, December 6.

1. Alan Turing was interested in modeling computations rather than accepting/rejecting inputs. His TMs had no Accept state. Given an input they either halted (which is good) or ran forever. So let $\mathcal{L}_{halt}$ = {(M,w) | M is a TM that halts (whether or not in a final state) on input w} If you prefer you can write this as {m111w | m is the encoding of a TM that halts on input w}. Show that $\mathcal{L}_{halt}$ is recursively enumerable but not recursive.

2. Describe informally (you don't need to draw it) a multi-tape TM that enumerates the perfect squares in the sense that it starts with blank tapes and prints on its first tape $0^1 1 0^4 1 0^9 1 0^{16} 1 0^{25} \ldots$

3. We showed that if a language and its complement are both RE then both are recursive. Suppose we have 3 recursively enumerable languages that are disjoint (no string is in two of them) and whose union is the set of all strings. Show that all three must be recursive.

4. Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are both recursively enumerable. Is the concatenation $\mathcal{L}_1\mathcal{L}_2$ RE? Why or why not?

5. We know $\mathcal{L}_{ne}$ is recursively enumerable but not recursive. Let $\mathcal{L}_{2ne}$ be {m | m encodes TM that accepts at least 2 strings} Rice's Theorem says $\mathcal{L}_{2ne}$ is not recursive. Is it recursively enumerable? Why or why not?

6. Let $\mathcal{L}_{inf}$ be {m | m encodes a TM that accepts infinitely many strings}. Is $\mathcal{L}_{inf}$ RE?